



# Teachers' Knowledge about Language in Mathematics Professional Development Courses: From an Intended Curriculum to a Curriculum in Action

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Received 5 December 2016 • Revised 15 January 2017 • Accepted 20 March 2017

## ABSTRACT

Explicit language objectives are included in the Swedish national curriculum for mathematics. The curriculum states that students should be given opportunities to develop the ability to formulate problems, use and analyse mathematical concepts and relationships between concepts, show and follow mathematical reasoning, and use mathematical expressions in discussions. Teachers' competence forms a crucial link to bring an intended curriculum to a curriculum in action. This article investigates a professional development program, 'Language in Mathematics', within a national program for mathematics teachers in Sweden that aims at implementing the national curriculum into practice. Two specific aspects are examined: the selection of theoretical notions on language and mathematics and the choice of activities to relate selected theory to practice. From this examination, research on teacher learning in connection to professional development is proposed, which can contribute to a better understanding of teachers' interpretation of integrated approaches to language and mathematics across national contexts.

**Keywords:** content and language integrated learning, curriculum implementation, knowledge about language, mathematics teachers, pedagogical content knowledge, professional development

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### **State of the literature**

- The importance of engaging pupils in oral practice for meaning-making is underlined in many studies on learning mathematics. A growing linguistic heterogeneity in classrooms has brought awareness of the need for literacy development as part of subject learning.
- To bring ambitions from an intended curriculum to a curriculum in action, subject teachers need specific knowledge about the language of the subject and practical skills.
- The design of professional development programs for content and language-integrated learning requires a focus on a specific language register, such as for mathematics. Previous research has identified required teacher skills, such as scaffolding students' language use.

### **Contribution of this paper to the literature**

- Professional development programs can function as starting points for examining teachers' role in the development of students' mathematics-language register in mathematics classrooms.
- The planning of teachers' professional development needs to be examined in terms of existing knowledge about teachers' competencies, particularly with a curriculum that aims to integrate students' language and mathematics learning.
- The design of professional development programs with a focus on the language in mathematics can be described as: a) the selection of relevant content, and b) teacher learning activities.
- Systematic description is meant to contribute to enabling comparisons of such programs despite their different national contexts.

## INTRODUCTION

The importance of academic language learning within content areas has been underlined in many studies and policy documents focusing on the education of migrant and second-language learners (OECD, 2010). Successful realization of ambitions in this direction, formulated at the national curriculum level, is a complex process that requires a multidisciplinary approach. Educational research can contribute to an understanding of teachers' role in curriculum implementation processes, while applied linguists and researchers in the field of mathematics education can contribute by selecting core content for professional development (PD) to build understanding of the processes of language and mathematics learning (van Eerde & Hajer, 2008).

Different intervention programs also address teachers' Professional Development (PD) for content and language-integrated approaches in multilingual classes (Vogt, Echevarria, & Short, 2010; Short & Echevarria, 2016). Despite the availability of subject-independent PD programs, mainstream content teachers often fail to identify with the role of providing language and literacy support to second-language learners in their classrooms (Davison, 2016; Hajer, 2006; Little, 2007; Norén, 2015).

Several factors have been proposed to explain these difficulties. First, the program may lack a subject-specific focus. Second, the relation between theoretical understanding and practice may be unbalanced, focusing too much on either of them (Hattie &

Timperley, 2007), or perhaps not making the connections between them sufficiently explicit. Third, crucial aspects of content and language-integrated teaching can be missed in such programs, as suggested in a Dutch study (Hajer, 2006), which identified the provision of feedback as important for both the language and content aspects of students’ utterances. Finally, the duration of PD programs and their components – explicit instruction, experimenting with new instructional tools, the provision of tutoring, etc. – would affect program outcomes (Short, 2013).

Teachers’ role in realizing a curriculum change is often taken for granted and remains a black box in large-scale efficacy studies. In his classical curriculum study, Goodlad (1979) distinguishes between different curriculum levels: the intended, the implemented, and the attained curriculum (Van den Akker, 2003, 2010, see Table 1).

In order to gain a better understanding of PD in the process of implementing language and mathematics integrated teaching, it is necessary to look closer at Goodlad’s dimensions at the stage in which teachers explore curriculum objectives in relation to their role in bringing the curriculum into the classroom. Even though this model may suggest a top-down perspective, teachers can be seen as autonomous professionals working within a given curriculum frame.

The aim of this project was to investigate: how PD programs can be designed to enable teachers to develop competencies for integrating language and mathematics learning. In addressing this question, we investigated a PD program that specifically aims to link an intended curriculum to an implemented curriculum, through enhancing mathematics teachers’ knowledge about the role of language in mathematics teaching and learning, as well as their skills to change mathematics classroom practices. A final aim for the PD, not investigated in this article, was the attained curriculum itself – the learning perceived by the learners and the outcomes of the changed practices.

**Table 1.** Typology of curriculum representations (Van den Akker, 2003, following Goodlad, 1979)

Intended	Ideal	Vision (rationale or basic philosophy underlying a curriculum)
	Formal, written	Intentions as specified in curriculum documents and/or materials
Implemented	Perceived	Curriculum as interpreted by its users (especially teachers)
	Operational	Actual process of teaching and learning (also, curriculum-in-action)
Attained	Experiential	Learning experiences as perceived by learners
	Learned	Resulting learning outcomes of learners

This article does not have the character of a typical research report. Empirically and theoretically we examined a specific PD program in Sweden that was developed to support teachers in realizing the official, intended curriculum that envisions a mathematic and language integrated curriculum (Skolverket [National Agency for Education], 2011). In its design, the program aims to bridge the space between teachers' theoretical understanding – in Goodlad's terms the 'perceived' curriculum, as interpreted by the teachers – and teachers' actual classroom practice – the 'operational' curriculum, in effect through teaching and learning. The issues of selecting relevant course content (Section 2) and handling the theory-practice dimension of the PD program (Section 3) will be discussed. The main part of this article (Section 4), examines the Swedish PD program. To set the scene, the background is described: a national, web-based program for primary and lower secondary teachers in Sweden, structured around collaborative team learning, of which the language and mathematics integrated module is a part. We will examine several parts of the module in more detail to find out how teacher learning in the PD program could be analysed in depth. In the concluding Section 5, we discuss how Goodlad's (1979) distinction between curriculum levels can help explain teachers' role in bringing curriculum changes to the classroom. Here we relate the selection of relevant knowledge about language with curriculum design principles for teacher learning activities (Bakkenes, Vermunt, & Wubbels, 2010).

Based on general considerations around the relationship between language and mathematics learning, and teachers' roles, the focus will be on the design of PD programs. In formulating a PD program focusing on language development in multilingual mathematics classes, the questions can be formulated as follows:

- How can specific knowledge, including know-how and concrete skills, about language in mathematics learning be included in a PD program that relates theory to practice?
- How can a PD program, explicitly designed to link theory and practice on language in mathematics, initiate and change teaching practices in mathematics?

In the concluding section, we will discuss how the specified outcomes of a teachers' PD program on language and mathematics can be studied systematically in the future.

#### SELECTION OF CONTENT IN PD

Choosing relevant course content in any PD program for mathematics teachers is a multifaceted endeavour. It requires the translation of findings on student learning and the language of schooling into teachers' practices. Characteristics of the language of schooling have been identified by scholars such as Schleppegrell (2004; Schleppegrell & O'Hallaron, 2011) and have been specified for mathematics where students have to mediate at least three linguistic registers: everyday language, the language of schooling, and the technical language of mathematics (Prediger, Clarkson, & Bose, 2016). For students with a mother tongue other than the language of instruction (even more obviously for newly arrived students), a disadvantage exists with respect to

listening skills, reading comprehension of written texts, and expressing themselves in written text and mathematical talk. The development of second-language skills at a high proficiency level can take several years after students' arrival (Short & Echevarria, 2004). During the last decade, aspects of subject-specific literacies have been included in national curricula, as in Australia and Sweden (Australian Curriculum and Reporting Authority (ACARA), 2015; Skolverket, 2011). Curriculum implementation nevertheless is highly dependent on teachers' understanding and interpretation of the language dimension as well as their skills in planning their lessons from a language- and content-integrated perspective. For classroom teachers who are teaching all subject areas in primary schools, this is not always obvious. Even more, content teachers at the upper primary and secondary level can be unaware of their potential role in language development for mathematics learning.

Worldwide, language diversity in classrooms has brought a growing awareness of the importance of PD for in-service teachers, including knowledge of the characteristics of subject-specific literacy and the role of language proficiency in content learning, and skills in including language development in their subject teaching. This has been referred to as Knowledge About Language (KAL) (Love & Humphrey, 2012), which can be considered a specific part of teachers' broader Pedagogical Content Knowledge (PCK) (Shulman, 1986; Hill, Ball, & Schilling, 2008). If teachers are well prepared, they can include academic language skills throughout the school year as a natural part of subject teaching (Lee, 2004). Thus, they can play a role in fostering students' language proficiency in mathematics.

Characteristics of KAL for mathematics teachers can be derived from linguistic analyses of mathematics textbooks, assignments, and classroom practices. Turkan, de Oliveira, Lee, & Phelps (2014) argue that knowledge about literacy aspects of different disciplines should be addressed in teacher preparation. Key factors in helping to produce a mathematically literate citizen are that reading and writing support students as they analyse, interpret, and communicate mathematical ideas, and as they interpret the validity of information and evaluate sources of information. Different scholars have taken steps towards creating such a knowledge base (Kilpatrick, Swafford, & Findell, 2001), drawing on different conceptualizations of mathematical literacy and particular social practices in the math classroom. Engaging pupils in oral practice for meaning-making is underlined in many studies on learning mathematics (Adler, 1998, 2001; Moschkovich, 2002, 2007, 2013). However, classroom observations in, for example, Quebec and Zimbabwe (Cleghorn, Mtetwa, Dube, & Munetsi, 1998) have shown limited active participation of students in mathematics classrooms. This has also been shown in Spain and the Netherlands, specifically, in multicultural classrooms with a greater linguistic heterogeneity (Civil & Planas, 2004; Deen, Hajer, & Koole (Eds.), 2008). These studies show that teachers play different roles in their support of students' development of content-specific literacy skills in mathematics (Österholm & Bergqvist, 2013).

Though the importance of pupils' inclusion in oral classroom communication is often highlighted, pupil participation in school mathematics is not only oral. Schleppegrell's

(2007) research review characterises meaning-making systems in mathematics as multiply semiotic: mathematics uses symbolic notations, oral language, and written language, as well as graphs and visual displays. Examining the grammatical patterns, she shows the characteristics of technical vocabulary, dense noun phrases, specific verbs, conjunctions with technical meanings, and implicit logical relationships (Schleppegrell, 2007, p. 142). Theoretical knowledge about language and language development, including metalinguistic terminology, is required for teachers in order to understand the rationale behind content- and language-integrated pedagogy; 'KAL is understood in a broad sense, encompassing any implicit or explicit reference to language, communication, and learning' (Arnó-Macià, 2009).

Apart from improving knowledge about specific register characteristics, training programs include practical skills in preparing subject- and language-integrated lessons. In general, PD for teachers of second-language learners includes three core issues in lesson planning (Vogt, Echevarria, & Short, 2010; Hajer & Meestringa, 2014; den Brok, van Eerde, & Hajer, 2010): being able to make new math concepts comprehensible and relate these to contexts that are familiar to students ('contextualization'); promoting active involvement in classroom interaction ('interaction'); and offering feedback and scaffolding with a specific focus on language use ('scaffolding' or 'feedback'). The definition and development of specific teaching skills for language- and math-integrated teaching has not been studied extensively. However, the importance of active participation in classroom interaction has been found in several studies of multilingual classrooms in different national contexts. These reports discussed the limitations of individual seatwork, for instance, and the importance of fostering interaction with learners during short moments of support from the teachers. The latter would require teachers' awareness of language, walking through the classroom, and using these short moments for individual, tailored scaffolding (Elbers, Hajer, Koole, & Prenger, 2008). Second-language learners, particularly, are dependent on active participation in classroom interaction and on planned content and language-integrated learning.

Specifications of teachers' various practices can be derived from classroom observations (such as Deen et al., 2008), which examine different teachers guiding pupils' learning (Den Brok et al., 2010). Around the world, from the 1990s onwards, many PD programs on content- and language-integrated learning with theoretical backgrounds in second- and foreign-language learning have been delivered, though mainly in secondary education and upper primary schools (Eurydice, 2006; Wisemann, 2008). Marsh, Mehisto, Wolff, and Frigols (2009) formulated teacher competencies in content- and language-integrated learning (CLIL). One widespread program is the SIOP approach to integration of academic language in content areas especially for adolescent second-language learners (Short, this volume). It structures PD around teacher steps in lesson planning, from formulating content and language objectives to building background, providing comprehensible input, supporting strategies, focusing on interaction, organising practice and application, lesson delivery, and planning for assessment. The approach has its roots in second-language pedagogy.

Before being able to measure the effects of PD programs on student learning, the learning outcomes of teachers as actors in curriculum implementation have to be studied. PD for teachers in content- and language-integrated teaching has been examined at a general level, not taking into account subject-specific pedagogies and pedagogical content knowledge. Hajer (2006) mentions the lack of subject specificity as a factor when discussing the success of PD programs in the Netherlands. For example, she mentions that the focus on reading strategies for longer texts and vocabulary did not fit mathematics teachers' needs as much as they met the needs of biology or history teachers, where other text types are part of the subjects' pedagogy. The same conclusion of a too-general approach to meet mathematics teachers' needs can be drawn for PD programs and materials available within Content and Language Integrated Learning programs (CLIL) in Wales and Kiribati (Coyle, 2009; Marsh, 2002).

Given the specific language requirements of mathematics, PD programs for mathematics teachers can be expected to include specified Knowledge About Language and well-selected skills for lesson preparation related to the identified register characteristics of the subject. Specific PD materials have been developed for teachers of a range of subjects, including a specific manual for teachers of mathematics (Vogt, Echevarria & Short 2010; see also Short, in this volume). Researching PD specified for different content areas can enable more insights to be found in the role of teachers in bringing language- and content-integrated curricula into the classrooms.

#### THE THEORY: PRACTICE DIMENSION OF PD DESIGN

Apart from their own selection of course content, teachers will be most influenced about the intended curriculum and the possibilities for putting it into action by the form and outline of PD programs.

Few studies have focused on teachers' learning about language in mathematics. Smit (2013) reports an educational design research study on one primary teacher learning to include math language in her lessons. She focusses on teachers' language scaffolding skills around one domain, interpreting graphs. In this domain, Smit shows the teacher's growing understanding of the mathematics register, putting this into explicit activities that foster pupils' metalinguistic awareness of differences between daily wordings and mathematical language by offering them well-prepared linguistic scaffolding. In this Smit follows the ideas of Schleppegrell (2007) and Gibbons (2009) about the importance of explicitly connecting and moving on the continuum between daily and academic registers in the functional context of doing mathematics in classroom practice. This type of qualitative research clarifies teachers' learning from selected knowledge about math language and math learning in connection to PD program characteristics, including supervision and reflection on classroom experiments through the use of videotaping.

Relating theory to practice is a general concern for PD; (Timperley, Wilson, Barrar, & Fung, 2008), in that teachers should find connections between course content and their own classroom routines. Hattie (2012) argues that continuous and systematic learning can occur where teachers and principals are encouraged to purposefully develop ways

of teaching grounded in research and proven experience in local schools. School contexts should include structured peer meetings/collegial interactions where teachers feel safe enough to reflect on strengths and weaknesses in their teaching and students' learning (Timperley & Phillips, 2003). According to Timperley and Phillips, teachers may not want to change and modify their teaching unless they believe that change will result in learning improvements. They propose an iterative changing process, wherein teachers' beliefs, actions, or teaching outcomes are built on each other. Established domain knowledge has to be challenged (Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005), and new domain knowledge has to be built.

Pedagogical Content Knowledge (PCK) (Shulman, 1986) is subject-specific in nature. It includes not only teachers' subject knowledge (mathematics) and generic pedagogical knowledge, but also topic-specific insights into what students think about, or how they can best be supported in their development of particular subject matter and skills (Hill et al., 2008). Joubert, Back, De Geest, Hirst, and Sutherland (2010) note that within school-based initiatives in Guyana focusing on general improvement of PCK in mathematics, reflective activities like group discussions or writing in a diary were important to the teachers. A similar result is found in Bakkenes et al. (2010), in a study on an innovation program for secondary teachers, who reported that learning occurred 'mostly through experimentation and reflection on their own teaching practices' (p. 544). Bakkenes et al. (2010) list possible learning outcomes of teachers participating in formal and informal learning environments as changes in knowledge and beliefs, intentions for practice, changes in practice, and changes in emotion. In their research, Bakkenes et al. relate learning outcomes to the types of learning that teachers have been exposed to. These can be formal, organised in classes and following a program, as well as informal. Four types of learning activities in PD programs are discerned:

- learning by experimenting (e.g., trying out instructional materials or scaffolding strategies),
- learning in interaction with others (other teachers, researchers),
- learning using external resources (e.g., publications), and
- learning by consciously reflecting on one's own teaching practice).

Further examination of these activities would enable a closer examination of their relationship to learning outcomes in PD programs.

In summary, PD programs that would enable teachers to integrate language development in their mathematics classes should

- include a subject-specific body of Knowledge About Language,
- offer different learning activities that relate theory to practice, and
- be organised in the setting of collaborative learning in school teams.

## BACKGROUND OF THE PD IN THE MATHEMATICS BOOST PROGRAM

The nationwide curriculum reform of 2011 in Sweden offers an interesting setting to construct the characteristics of a PD program against the literature review presented



above. First a general background to the PD program is given, and then the PD module on language in mathematics will be examined.

### **A (language) Perspective on Swedish Mathematics Education and Teachers' Intended Learning**

The PD program, called 'Mathematics Boost' (Matematiklyftet), should clearly be seen from the perspective of the National Curriculum for the Compulsory School, introduced in 2011 (Skolverket, 2012). Mathematical communication is highlighted in this new syllabus, which aims to educate students in the exchange of mathematical ideas and thoughts with others. Long-term goals state that students should be given opportunities to develop the ability to formulate problems, use and analyse mathematical concepts and relationships between concepts, lead and follow mathematical reasoning, and use mathematical expressions to discuss, argue, and explain the issues, calculations, and conclusions (Kilpatrick et al., 2001). Explicit work on language proficiency is therefore essential for students to achieve the curriculum's long-term goals in mathematics (Adler, 1998).

In order to implement this curriculum in the classroom, teachers should understand it and be willing and able to translate it into practice (Goodlad, 1986). The observation that students' individual seatwork is more usual in Swedish mathematics classrooms (i.e., substantial time is spent in silence) was a strong incentive for the Swedish government to implement a mathematics teachers' PD program. Research had shown the need for a more communicative, interactive mathematics education (Kilpatrick et al., 2001). Already in 2004, the Swedish National Agency for Education noted that the total time students spent working independently in mathematics textbooks had increased (Skolverket, 2004). The amount of time that teachers instruct an entire group has declined in the last 20–25 years. In 2004, approximately 6 per cent of the time in Swedish mathematics classrooms at all levels was devoted to 'inquiry based' mathematics and laboratory practices where more conceptual than procedural learning could be applied (Skolverket, 2004). Liljestrand and Runesson (2006) explored how classroom organisation, tasks, and content shape the interaction as well as learning potential, and showed that classes typically began with an introductory plenary session that was followed by individual seatwork from a textbook. These studies of mathematics education uncovered the minor role that teachers played in actual classroom interaction, while students increasingly worked on their own with mathematics books. Several other researchers have noted this relationship (Kling Sackerud, 2009; Sjöberg, 2006; Österholm & Bergqvist, 2013).

The National Agency for Education plays a steering role, requiring schools to arrange PD, using the kit of PD materials as a condition for getting funding for PD, and providing training for supervisors and tutors leading the PD. At the same time, an active role of teachers in their own learning is expected, drawing on Hattie and Timperley's recommendations for collaborative learning in teams of teachers (Schnellert, Butler, & Higginson, 2008). The content of the PD program is structured around students' collaborative and interactive learning, and teachers are expected to

learn within the context of communities of practice with colleagues in their own school. An assumption is that teachers' interactive and collaborative learning should start with the teachers bringing a substantial body of knowledge into the collaboration.

### **The mathematics Teachers' Professional Development Program**

The Swedish government decided to spend a total of 649 million crowns (roughly 76 million USD) starting in the school year 2012/13 and continuing for three additional academic years so that all teachers who teach mathematics within the school system would be able to participate in Mathematics Boost. The funds were used for program development and support for schools, for example in the form of compensated hours for teachers to participate. In addition, tutors – specialised mathematics teachers – were educated at different universities to lead and support the teams of teachers using Mathematics Boost in their schools.

The PD material is published for mathematics teachers as a national, web-based program with didactical support material ([www.matematikportalen.se](http://www.matematikportalen.se)). The National Agency for Education consulted with universities' and colleges' mathematics education staff members, who were assigned to create the web platform content. The construction of web-based materials can be seen as a wider pathway in contrast to PD that involves off-site activities, where physical attendance can become an impediment. The main materials on the web platform are training packages, called modules, which teachers are supposed to work through collaboratively in planned sessions.

In addition to providing teachers with professional development, the overarching aim of Mathematics Boost is to increase students' achievement in mathematics through the strengthening of mathematics teaching (Skolverket [National Agency for Education], 2012). In other words, the purpose of the program is to influence two processes, namely the mathematics classroom practices (the teaching, of which working with language development is one aspect), and the professional development culture, to engage teachers in processes of collective learning where they relate new knowledge to their classroom experience. The construction of the PD program leans heavily on an assumption that mathematics teachers are considered to be in need of Continuing Professional Development (CPD) (Joubert & Sutherland, 2009) in order to implement the intended curriculum (Goodlad, 1979, 1986). Even though Swedish teachers may have participated in collaborative learning before, Mathematics Boost strongly articulates this as a way to develop teachers' teaching. State funding of teachers' participation requires schools to follow the framework and learning activities. Thus, the PD program supports collegial learning in communities of practice (Wenger, 1998), in which colleagues' structured collaboration aims to integrate new knowledge into day-to-day practices (Smit, & van Eerde, 2011). In the program, participating teachers work with the various modules consisting of didactic materials to use for discussing, planning, and evaluating mathematics teaching.

Modules typically consist of eight parts, which are meant to be the focus for one school term (20 weeks), during which all teachers spend two hours a week for a total of 40

hours. The fixed format for each part is meant to structure the collaborative work of teachers. Each part consists of four sections called A, B, C, and D.

Section A is an individual preparation for each teacher, who reads an introductory article and/or watches a video clip relating to the parts' theme. This would take about 45–60 minutes. Section A represents the intended curriculum, including the theory (Goodlad, 1979, 1986). Section B is related to Goodlad's perceived curriculum: in a meeting, teachers discuss the literature and video, aided by a number of focus questions and led by a tutor or supervising teacher. From these discussions, practical applications of didactical ideas in the teacher's own classroom is prepared (90–120 minutes). Section C is then the actual classroom activity that is part of the ordinary classroom work of each teacher. Thus, Section C forms the curriculum in action (Goodlad's operational curriculum. Section D consists of another group meeting where the teachers reflect on their experiences with class activities and draw conclusions about the part-theme (45–60 minutes), thus relating the implemented curriculum to perceived outcomes, the attained curriculum (as illustrated in Table 1).

Within a module, learning activities for the teachers are always repeated in these four-cycle sections: a) individual studies/work: read an article and/or watch a film sequence; b) group discussion on the articles and films, and plan lesson collaboratively; c) conduct lesson in one's own class/group, observe other teachers' teaching; and d) group discussion, follow-up, and collaborative evaluation of the conducted lesson.

By the end of 2015, about 14,000 teachers across the country had gone through a year of the Mathematics Boost program (Jahnke, 2015; Ramböll, 2015). In the summer of 2016, 76 per cent of all mathematics teachers in Sweden had participated in the program to various extents (Skolverket, 2016b). This translates into 35,580 teachers.

## THE MODULE ON LANGUAGE DEVELOPMENT IN MATHEMATICS: DESIGN AND CONTENT

### General Description of Course Content and Learning Activities

In May 2016, eight PD modules for compulsory school and seven for upper secondary school had been developed, many of them focussing on specific content areas like 'graphs', 'arithmetic', or 'geometrical forms'. The module we focus on in this article has a more general focus, 'Language in mathematics' for compulsory school. The module targets mathematics teachers working with pupils in the age range of 7–16. Different disciplines were represented when constructing the module: mathematics pedagogy, educational linguistics, and second-language learning. All materials were peer-reviewed in two cycles by the National Agency for Education, researchers, and teachers before being published on the open-access website.

Through this module, teachers should realize that students' oral and written communication is essential for learning mathematics. To enhance classroom communication for students' mathematical learning, a major part of the module was to establish teacher practices that take into account reading mathematical texts, and writing such texts (Österholm & Bergqvist, 2013). Thus, this module for teachers of

mathematics was developed to prepare for designing and delivering lessons under the intended curriculum, in which an explicit focus is on students' language development in mathematics.

The following is an overview of this module that presents the content and provides references to didactical models. We then describe in more detail three parts (3, 4, and 6) that have been mentioned by teachers in practice to be the most fruitful for their work (Norén, Ramsfeldt, & Österling, 2016). We also account for the attained curriculum in an activity in school year 5, conducted by a teacher who participated in the PD program, using videotaped data. The video has been shortened and published on the web for other teachers to view when working with Part 6. We go on to describe and analyse the selected knowledge about language included, the link to mathematics content, and the learning activities that aim to link theory to teachers' classroom practice, formulating the potential learning trajectories. Table 2 presents an overview of the modules' eight parts, its content focus, didactical models, and learning activities.

In Part 3, 'Communication from a formative perspective', the practice of two teachers in their actual classroom interaction is compared using original classroom transcripts derived from Deen et al. (2008) to show how teachers' daily practice can be more or less supportive for language development. In Section A, the learning activities for the teachers start with an introductory text directly linking the knowledge about language to a specific content area (Schlepppegrell, 2004): reading and understanding graphs (van Eerde & Hajer, 2008). The importance of hearing students' thoughts in order to adapt teaching to their prior knowledge and existing language skills is a key aspect of Part 3. In addition, working with concept maps is introduced as a practical activity (also in Section A), starting from a list of relevant key terms from the mathematics syllabus. Concept maps visualize and make explicit the relationships between words and the required connecting words. In a concept map, a focus question is formulated that organises conceptual knowledge (Novak, 1990). Planning lessons in which concept maps are used to visualize prior knowledge, or to elaborate on new course content, are suggested in Section B. Section C (delivering the planned lesson) includes the gathering of students' maps, which stimulates teachers to promote students' active use of language and thus grasp students' prior knowledge at the beginning of a new mathematical unit. In this way, the crucial step of formulating language objectives in math lessons is presented. Teachers explicitly have to emphasise concepts like charts, graphs, line, curve, rise, fall, and line charts. In addition to the individual terms, the relations between the terms and descriptions of those relationships need to be considered. For the development of language, an overall plan for progression is suggested: a) The relationship between simple graphs and daily phenomena can be discovered in conversations, for example in discussing the temperature or distance. b) In small groups, students discuss the different graphs, identify features, and make comparisons. The teacher listens, supports by paraphrases, and shares formal words. c) Students draw and interpret graphs, and relate them to functions. They present results and are expected to express themselves with mathematical language. In Section D, the teachers' individual lessons are evaluated and then discussed with colleagues.

**Table 2.** Overview of the module, including the content and reference to didactical models

<b>Parts</b>	<b>Content Focus</b>	<b>Key reference</b>	<b>Learning activities</b>
	Section A	<b>to didactical models</b>	Section C
1. Language- and content-knowledge approach	Characteristics of language in mathematics teaching and learning	Three principles for sheltered instruction (Vogt et al., 2010) and CLIL	Plan, carry out, and evaluate lesson, with relevance for principles 1 and 2 of the 3 principles for CLIL
2. The mathematical language	The mathematics language's segments and their relation to other genres	Representations, a 'thinking board' or matrix (McIntosh, 2006), in which students draw and write four different mathematical representations: picture, material artefacts (hands-on material, manipulatives), symbols, words	Plan, carry out, and evaluate lesson: What do students know about informal and formal words and symbols in the mathematics register?
3. Communication with formative purposes	Connecting to students' prior mathematical knowledge. Content focus on graphs and their representation	Concept maps, (Novak, 1990)	Plan, carry out, and evaluate lesson: Construct conceptual map on current teaching of mathematics content
4. Scaffolding the mathematics language	Scaffolding language. Content focus on dynamic geometrical program, sequencing, visualization, reformulation, contrast	Micro (interactional) scaffolding, macro scaffolding (Hammond & Gibbons, 2005)	Plan, carry out, and evaluate lesson: Macro scaffolding in relation to the current teaching of mathematics content

**Table 2.** continued.

Parts	Content Focus	Key reference	Learning activities
	Section A	to didactical models	Section C
5. Interaction in the mathematics classroom	How to put questions to students, various questions, open questions in classroom interaction	IC model (Inquiry & Cooperation) (Alrø & Skovsmose, 2004)	Plan, carry out, and evaluate lesson: Student interaction around the current teaching of mathematics content
6. The teaching learning cycle: Text tasks in mathematics	Analyses of mathematical text problems	The teaching learning cycle, (Gibbons, 2002; Derewianka, 2003)	Plan, carry out, and evaluate lesson: Text problems in the current teaching of mathematics content, analyses together with students
7. To produce texts in mathematics	Writing and production of mathematical texts	The teaching learning cycle (Derewianka, 2003)	Plan, carry out, and evaluate lesson: Construct text problems together with students
8. Reflecting and looking forward	How one's own teaching in mathematics has developed, and can be more developed, regarding language development in mathematics	Metacognition, reflection	Discuss and evaluate with colleagues

Part 4 focuses on 'Scaffolding language in mathematics'. Offering students various opportunities to communicate mathematics is at the forefront. To communicate in the mathematics classroom means to exchange information with others about mathematical ideas and thoughts, orally and in writing, using different forms of expression (Love & Humphrey, 2012). In teaching, students have the opportunity to develop a more precise mathematical language to independently adapt their talks and presentations to various recipients or purposes. As key knowledge about language, the concept of 'scaffolding' (Gibbons, 2002; Hammond & Gibbons, 2005) is foregrounded.

After watching a film (Section A) from a classroom in which the mathematics teacher macro- and micro-scaffolds a whole class or students in pairs at different occasions, teachers are encouraged to give examples of how they already scaffold students in their own teaching. In Section B, the teachers discuss their individual experiences collectively and plan a lesson collaboratively to be delivered in each teacher's class (Section C). Teachers are asked to relate their lesson to sequencing, reformulation, visualizing, and contrasting (Hammond & Gibbons, 2005). In Section D, individual evaluations of the lessons are discussed.

Part 6 introduces the teaching-learning cycle for elaborating on texts in mathematics. Throughout Sweden, the generic Australian pedagogy had reached many classrooms. In this approach (Rose & Martin, 2012; Gibbons, 2002), an explicit focus on written texts is introduced in mathematical content areas. Even within the profession of mathematics, its texts have specific challenges for readers and writers. Therefore, it is argued, teachers should explicitly talk with students about the characteristics of a 'good' mathematics text and practice writing such texts themselves (Rezat & Rezat, 2017).

#### **Part 6, Work on Mathematics Texts, through Sections A-D: An Illustration**

The individual preparation for each teacher in Section A includes reading an introductory article, 'The teaching and learning cycle: Texts in mathematics', that discusses how teachers can practically work with language development in mathematics instruction using the teaching and learning cycle. It can be considered a clarification of the intended curriculum (Goodlad, 1979). A reflecting question for the teachers to individually consider is: What are your experiences of paying attention to your students on the language, structure, and context of different texts in mathematics? Teachers are told to prepare for Section B by selecting a few mathematical texts from the mathematics topic they are currently working on in their own classes. They have to justify their choices based on experiences with reading the article. The selection will serve as a basis for analysis of mathematical texts' characteristics in terms of language, structure, and context. Some texts will also be chosen as typical examples, and the teachers will discuss what makes the chosen tasks interesting to analyse in terms of language, structure, and context.

Teachers write down their reasoning about the issues above and bring examples of texts to the collegial work in Section B. Section B is related to Goodlad's (1979) perceived curriculum: the teachers meet and discuss the article and their chosen mathematical texts. They also watch a three-minute video clip of a Swedish teacher talking about the teaching and learning cycle. Aided by the focus question from Section A, and led by a supervising teacher, the discussions serve as preparation for practical applications of didactical ideas in the teachers' own classrooms. The teachers are instructed to analyse one or two mathematical texts, with respect to language, structure, and context.

Section C is the actual classroom activity. Thus, Section C forms the curriculum in action, or the implemented curriculum, relating to Goodlad. The video (Skolverket,

2016a) we discuss below is part of a recorded lesson, and shows an example of implementing the analysis of mathematical texts in the classroom.

Finally, Section D consists of a second meeting (45–60 minutes), where teachers reflect on their experiences with classroom activities and draw conclusions about the theme, thus relating to the attained curriculum. Focus questions are:

#### Evaluate

Did the lesson fulfil its purpose? What helped to make it fulfil the purpose? What obstacles did you experience?

- What aspects of the organisation of the lessons worked well?
- What worked less well? Why?
- In what ways did you adapt scaffolding during the lesson?
- How do your experiences of classroom activities differ?

#### Reflect

- In what way do you think the teaching and learning cycle supported the lesson planning and implementation?
- Which ways promoted in the lesson helped students develop mathematical language?
- How would you be able to jointly develop student skills in analysing the language, structure, and context of other types of texts in mathematics that they will encounter and produce in mathematics?
- How could you work with other kinds of texts in mathematics using the support of the teaching and learning cycle?

### **Part 6, Video: One Lesson in School Year 5 (students 11–12 years of age)**

The aim of the lesson is to make students aware of mathematical and everyday vocabulary in a written mathematical text, but also to extend their vocabulary generally. In the introduction to the lesson, the teacher tells her students that she has moved and her way to school is now a longer distance from home than before. The students are invited to talk about their own way to school. The students animatedly describe how they walk, bike, or are driven to school by their parents. They talk about hilly and flat parts on their way, and how talking to other students along their way might make them walk at a slower pace. After about eight minutes, the teacher introduces a mathematical text that she and the students will analyse together, in line with the teaching learning cycle (Gibbons, 2002):

Early one Tuesday morning, Pelle rides his bike to school. He maintains a high average speed until half the distance to school is covered. There at the big oak, he stops and waits for Fia. Suddenly he realizes that he should have fetched Fia at her house. He rides back two-thirds of the stretch of road he had already cycled, at the same speed as he had before. After a short waiting time when Fia unlocks her bike, they ride together



to school. They talk so their speed is only half of what Pelle's speed was before. When they are halfway to the big oak from Fia's house, Pelle looks at the clock and sees that now they need to hurry. They increase their speed so that they ride twice as fast as Pelle cycled, from the beginning, the rest of the way to school. They arrive on time.

The teacher starts by telling the students to underline the words in the text that they find a bit unusual and difficult to understand, and she goes on reading the text aloud.

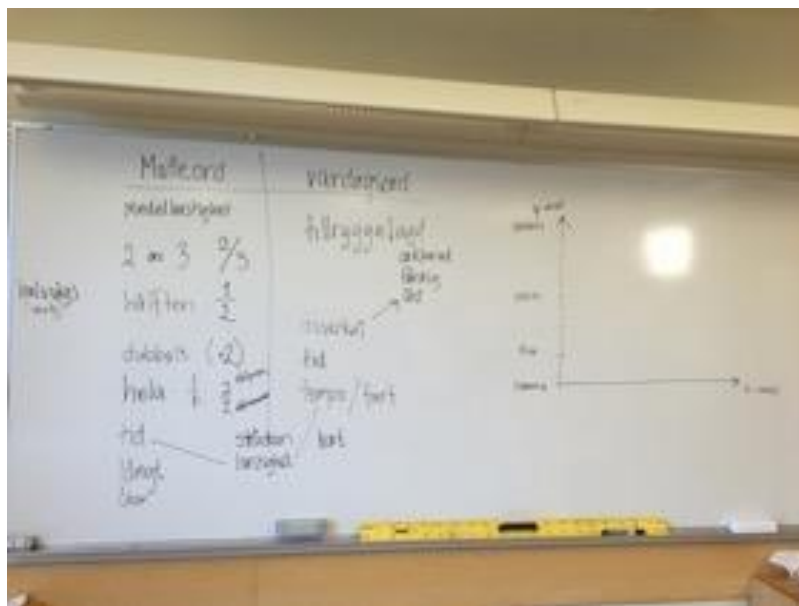


**Figure 1.** A student is underlining unusual words

The first word underlined comes from Jacob, who says *tillryggalagd*. The translation to English is distance travelled, but what it really means is distance you put behind your back. On the whiteboard, the teacher has written headings for two columns: mathematical words and everyday words. The students are invited to talk about the word. Students make suggestions like: he has already done it; he has it behind himself. Other suggestions for explaining the word include put behind the back, completed, ready. The students agree that the word is an everyday word, arguing that it is not mathematical. More words are discussed, elaborated on, and defined: average speed [*medelhastighet*], a mathematical word; two-thirds (*två tredjedelar*, two out of three), mathematical words; half [*hälften*], a mathematical word; the distance [*sträckan*], a word that can be both mathematical and used in everyday language. At the end of the 60-minute lesson, the whiteboard looks like in Figure 2:

The next lesson covers drawing a graph. On the y-axis it says: home, Fia, oak, school. The x-axis is the time.

The 'intended' curriculum in the national curriculum states, Students have to develop their ability to conduct mathematical reasoning, and as stated in the PD program in a more operationalized way, Texts in mathematics present challenges of different kinds for the students. Elaboration on various types of mathematical texts anchors and deepens students' knowledge, both in terms of the mathematical content and mathematical language. The 'attained' curriculum, what students experience and learn, is exposed in the lesson, a lesson that is the teacher's 'implemented' curriculum.



**Figure 2.** The writings on the whiteboard at the end of the lesson

## DISCUSSION

In the discussion, we expand from the research questions: on specific knowledge, in a PD program, about language in mathematics, and how the knowledge can include know-how and concrete skills; on how the design of a PD program can link theory to practice by initiating and changing teaching practices in mathematics; and on the specified outcomes of PD programs and how they can be studied in the future.

### **Specific Knowledge on Language in Mathematics, Teachers' Learning, and PD Programs**

The language module in the larger Swedish Mathematics Boost PD program offers an interesting example that meets the requirements of subject-specific Knowledge About Language. The PD program for mathematics teachers does not impose a language perspective onto their teaching role, but enlightens the language dimension as a natural part of mathematics subject content and pedagogy. The scale on which the program is spread throughout Sweden, and the similar conditions of the structured ABCD sections, offer possibilities for a closer examination of teacher learning, putting the intended curriculum into practice. In staff meetings, for instance, teachers could discuss the relevance of theoretical concepts (Parts A and B), illustrate how they use and develop skills in the classroom (Part C), and reflect upon their experiences (Part D). Up to now, evaluation studies of the program have been large-scale and focussed on appreciation of the PD structure, its setting in collaborative meetings, and time factors. One study showed 6,000 teachers' appreciation of the courses and examined conditions for its web-based nature of in-service training in combination with

collaborative learning contexts in school teams (Ramböll, 2015). The average outcomes show a positive rating of the material's relevance for teaching mathematics. One of the main factors in teachers' judgments is the content and structure of modules. The report judges the conditions for realization offered by the National Agency for Education as 'good'. One recommendation is that more flexibility should be enabled in using parts of modules and adapting the in-service portion to specific needs in the school team. The report did not examine teachers' learning within PD around specific modules. Further examination could discern the role of diversity within school contexts, teachers' individual development, and students' needs at different stages of learning.

### **The Design of the PD Program and Teachers' Change of Practice**

Concerning the learning activities chosen, we can see that each part in a module consists of four sections: a) individual studies/work: read an article and/or watch a film sequence; b) group discussion on the articles and films, and plan lessons collaboratively; c) conduct lesson in one's own class/group, reflect on own teaching; and d) group discussion, follow-up, and collaborative evaluation of the conducted lesson. The design of the PD matches Loucks-Horsley, Stiles, Mundry, Love, and Hewson's (2009) professional development design framework, building on reflection about and revision of teaching. Considering the four teacher/learning activities listed by Bakkenes et al. (2010), we find each of them in the different sections of the module. Each Section C focuses on learning by experimenting (trying out instructional materials). Each Section B and D contains learning in interaction with others (recurring collegial discussions with other teachers). In the preparation, teachers are asked to read and watch external resources (film and article). Within Sections A, B, C, and D, consciously reflecting on one's own teaching practice is promoted, and it is proposed to do this collaboratively.

Apart from the recurrent, theory-practice linking of activities, a strength of the PD program is that it fits closely with the intended national curriculum on mathematics and pedagogical context and traditions in Swedish schools, likely due to the active involvement of the National Agency for Education. The materials reach schools not as part of a language pedagogy PD program, but as part of the national Mathematics Boost program addressing mathematics teachers' concerns, and through relating explicitly to the mathematics curriculum and syllabus guidelines. The content of the PD program helps teachers to implement the curriculum. The fact that authors of the materials worked in a multidisciplinary team has contributed to the program's mathematics-specific nature. What is more, through the process of designing the module, in which experts and practitioners were actively involved, the present materials can be seen as a state-of-the-art example of the Swedish idea regarding relevant knowledge about mathematics language.

Even though the module is not specifically constructed for schools with second-language learners only, it has its roots in research on second-language learners and content-based instruction. It is aimed at teachers' awareness of learning, teaching, and ways of using language/s in relation to a specific school subject (Marsh, 2002). We can

discern, in the module's core knowledge, a strong focus on language development in interaction, characteristics of mathematics language (in vocabulary as well as text types), and a strong focus on teacher scaffolding and feedback in interaction. Given the high number of newcomer students in the Swedish schools, one remaining question is whether the module provides sufficient understanding of the specific second-language pedagogy aspects required in newcomer classes.

In its actual use, the PD program may look different because the teachers and their tutors choose mathematics content from their daily teaching or textbooks. Here, we can expect major differences in teacher practices. The PD program is being used throughout the country at large, reaching hundreds of schools and thousands of teachers in different contexts. Group tutors leading the team work within the PD could certainly adapt the program to current concerns of teachers, be it addressing the needs of newly arrived pupils or including pupils in group work, just to mention two examples. The realization of the PD will therefore look different in different contexts, as will the outcomes.

One can raise the question of how the pedagogical tools offered in the module can be used as part of a comprehensive approach of teaching within a thematic unit or a certain area of mathematics. If we compare the program to the SIOP approach (Short & Echevarria, 2004), no comprehensive planning tool is offered, from introduction of new concepts and terminology to assessing student learning at the end of a unit (Hajer, 2006). It would be interesting to see how teachers take up and include the suggestions in their daily routines and planning.

### **Studying the PD for Language in Mathematics**

In examining the design and content of PD for language in mathematics, desired outcomes have to be described in national settings (the attained curriculum). In Sweden, language learning has been integrated as an aspect of effective mathematics education that offers opportunities for all learners in multilingual classrooms. The design of the PD content and delivery were organised as a transparent, nationwide effort, made public through web-based materials. However, there is a tension between the need for uniformity and large-scale PD and more tailor-made programs adapted to specific school contexts, generating a requirement for comparative studies on strengths and weaknesses of PD programs. We argue that there is an absolute need for better understanding of teachers' work in constructing the syllabus, which should foster a communicative mathematics education in multilingual classrooms. Using Goodlad's (1986) distinction between the curriculum aimed for, interpreted, and realized, teachers are of crucial importance in understanding, being willing, and being able to plan and deliver their mathematics teaching in line with the national curriculum guidelines. In order to understand how they do this and develop new routines, the delivery of the Mathematics Boost PD offers an opportunity to gather data and compare teachers' learning in collaborative groups, taking pedagogical activities to their classrooms and reflecting on the outcomes.

There is a need for a better understanding of teachers' learning and the quality of their development of know-how and skills about language in mathematics. Davison (2016) states that many mainstream teachers fail to identify with the role of providing language and literacy support to second-language learners in their classrooms. To address this issue, Hammond (2014, p. 503) calls for 'more wide-ranging, theoretically robust accounts of teacher learning' to specifically support these learners. Our way of describing PD course content in general terms, subject specificity and the learning activities are meant to contribute to enabling comparisons of research on PD in various national contexts. If PD programs could be described in a similar way, the next step in creating a rich knowledge base would be to synchronize data gathering. In order to compare the PD programs in a systematic way, a better description of content and types of learning activities and synchronized assessment of learning outcomes would be required. Selected Knowledge About Language, chosen learning activities as well as achieved changes in knowledge and beliefs, changes in intentions for practice, and changes in actual practice are all relevant. If these categories could be described in similar ways, different PD programs and contexts could be compared within an international perspective, thus deepening our understanding of their effectiveness. In future research on PD we propose to examine how Simon's construct of Hypothetical Learning Trajectories (Simon & Tzur, 2004; Simon, 2014) could be of help. Explicating Hypothetical Learning Trajectories for teachers' learning and bringing theory to practice in selected parts of PD programs could enable a closer examination of teachers' learning. Although the authors of Mathematic Boost materials did not explicitly formulate such hypotheses, researchers as part of their evaluation studies could formulate them.

We argue that formulating underlying assumptions about fostering teachers' roles in students' language development in mathematics is a prerequisite for further studies on teacher PD as the crucial link in implementing language- and mathematics-integrated curricula.

## REFERENCES

- Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics*, 18(1), 24–33.
- Adler, J. (2001). *Teaching Mathematics in Multilingual Classrooms*. Dordrecht: Kluwer Academic Publishers.
- Alrø, H., & Skovsmose, O. (2004). *Dialogue and learning in mathematics education: Intention, reflection, critique* (Vol. 29). Dordrecht, Boston, New York, London *Dialogue and learning in mathematics education: Intention, reflection, critique*: Kluwer Academic Publishers.
- Arnó-Macià, E. (2009). Knowledge about language in English language Courses for future language professionals. *Nordic Journal of English Studies*, 8(1), 5–39.
- Australian Curriculum and Reporting Authority (ACARA) (2015). *The Shape of the Australian curriculum, version 4*. Retrieved the 15<sup>th</sup> of August from <http://www.acara.edu.au/curriculum>

- Bakkenes, I., Vermunt, J. D., & Wubbels, T. (2010). Teacher learning in the context of educational innovation: Learning activities and learning outcomes of experienced teachers. *Learning and Instruction* 20, 533–548. DOI:10.1016/j.learninstruc.2009.09.001
- Civil, M., & Planas, N. (2004). Participation in the mathematics classroom: Does every student have a voice? *For the learning of mathematics* 24(1), 8–14.
- Cleghorn, A., Mtetwa, D., Dube, R., & Munetsi, C. (1998). Classroom language use in multilingual settings: mathematics lessons from Quebec and Zimbabwe. *International Journal of Qualitative Studies in Education*, 11(3), 463–477. DOI:10.1080/095183998236610
- Coyle, D. (2009) Promoting cultural diversity through intercultural understanding. A case study of CLIL professional development at inservice and preservice level. In M. Carrió-Pastor, *Content and language integrated learning: cultural diversity*, pp. 105–124. Bern: Peter Lang.
- Davison, C. (2016). Collaboration between English language and content teachers: Breaking the boundaries. In A. Tajino, T. Stewart, & D. Dalsky (Ed.), *Team Teaching and Team Learning: Collaboration for Innovation in Language Classrooms*, pp. 51–66. New York: Routledge.
- Deen, J., Hajer, M., & Koole, T. (Eds.). (2008). Interaction in two multicultural mathematics classrooms: Mechanisms of inclusion and exclusion. Amsterdam: Aksant.
- den Brok, P., van Eerde, D., & Hajer, M. (2010). Classroom interaction studies as a source for teacher competencies: the use of case studies with multiple instruments for studying teacher competencies in multicultural classes. *Teachers and Teaching: theory and practice*, 16(6), 717–733. DOI: 10.1080/13540602.2010.517689
- Derewianka, B. (2003). Trends and issues in genre-based approaches. *RELC Journal*, 34(2), 133–154. DOI: 10.1177/003368820303400202
- Elbers, E., Hajer, M., Koole, T., & Prenger, J. (2008). Instructional dialogues: participation in dyadic interactions in multicultural classrooms. In J. Deen, M. Hajer, & T. Koole (Eds.), *Interaction in Two Multicultural Mathematics Classrooms. Mechanisms of Inclusion and Exclusion*, pp. 141–171. Amsterdam: Aksant, NWO-reeks.
- Ferrini-Mundy, J., Floden, R., McCrory, R., Burrill, G., & Sandow, D. (2005). *Knowledge for teaching school algebra: Challenges in developing an analytic framework*. American Education Research Association. Montreal, Quebec, Canada.
- Gibbons, P. (2002). *Scaffolding language, scaffolding learning: teaching second language learners in the mainstream classroom*. Portsmouth, NH: Heinemann.
- Gibbons, P. (2009). English learners, academic literacy, and thinking. *Learning in the Challenge Zone*. Portsmouth, England: Heinemann.
- Goodlad, J. I. (1979). *Curriculum inquiry. The study of curriculum and practice*. New York, McGraw Hill.
- Goodlad, J. I. (1986). The learner at the world's center. *Social Education*, 50(6).
- Hajer, M. (2006) Inspiring teachers to work with content-based language instruction – stages in professional development. In I. Lindberg & K. Sandwall (Eds.) *Språket och kunskapen – att lära på sitt andraspråk i skola och högskola*, pp. 27–46. Gothenburg: Institutet för Svenska som Andraspråk, Rosa-serien.
- Hajer, M., & Meestringa, T. (2014). *Språkinriktad undervisning: en handbok*. Stockholm: Hallgren & Fallgren.
- Hammond, J. (2014). An Australian perspective on standards-based education, teacher knowledge, and students of English as an additional language. *TESOL Quarterly*, 48(3), 507–532. DOI: 10.1002/tesq.173

- Hammond, J. & Gibbons, P. (2005). Putting scaffolding to work: The contribution of scaffolding in articulating ESL education. *Prospect*, 20(1), 6–30.
- Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. London, New York: Routledge.
- Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research*, 77(1), 81–112.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372–400.
- Jahnke, A. (2015). Hur högt kan ett lyft ta oss? *Pedagogiska Magasinet*, 3.
- Joubert, M., & Sutherland, R. (2009). A perspective on the literature: CPD for teachers of mathematics. National Centre for Excellence in the Teaching of Mathematics. Retrieved from [https://www.ncetm.org.uk/files/387553/RECME\\_Literature\\_Review.pdf](https://www.ncetm.org.uk/files/387553/RECME_Literature_Review.pdf)
- Joubert, M., Back, J., De Geest, E., Hirst, C., & Sutherland, R. (2010). The researching effective CPD in mathematics education (RECME) research project. *Research in Mathematics Education*, 12(1), 77–78.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Research Council. Mathematics Learning Study Committee.
- Kling Sackerud, L. A. (2009). *Elevers möjligheter att ta ansvar för sitt lärande i matematik: En skolstudie i postmodern tid* (Doctoral dissertation, Institutionen för matematik, teknik och naturvetenskap, Umeå universitet).
- Lee, O. (2004). Teacher change in beliefs and practices in science and literacy instruction with English language learners. *Journal of Research in Science Teaching*, 41(1), 65–93. DOI: 10.1002/tea.10125
- Liljestrand, J., & Runesson, U. (2006). Interaction, organisation, tasks and possibilities for learning about mathematical relationships: A Swedish classroom compared with a US classroom. In D. Clarke, E. Emanuelsson, E. Jablonka, & I. A. Chee Mok (Eds.), *Making connections: Comparing mathematics classrooms around the world*, pp. 165–184). Rotterdam, Taipei: Sense Publishers.
- Little, D. (2007). The common European framework of reference for languages: Perspectives on the making of supranational language education policy. *The Modern Language Journal*, 91, 645–655. DOI:10.1111/j.1540-4781.2007.00627\_2.x
- Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2009). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin Press.
- Love, K., & Humphrey, S. (2012). A multi-level language toolkit for the Australian Curriculum: English. *Australian Journal of Language and Literacy*, 5(2), 173.
- Marsh, D. (2002). Language awareness and CLIL. In J. Cenoz & N. H. Hornberger (Eds.), *Encyclopedia of Language and Education*, 2<sup>nd</sup> Edition, Volume 6: *Knowledge about Language*, pp. 233–246. New York: Springer.
- Marsh, D. P., Mehisto, D., Wolff, M. J., & Frigols, M. (2009). *European framework for CLIL teacher training*. Strasbourg: Council of Europe.
- McIntosh, A. (2006). Nya vägar i räkneundervisningen. In J. Boesen (Ed.), *Lära och undervisa matematik – internationella perspektiv*. Göteborg: NCM.

- Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. In N. Nassir & P. Cobb (Eds.), *Mathematical Thinking and Learning, Special issue on Diversity, Equity, and Mathematical Learning*, 4(2&3), 189–212. DOI: 10.1207/S15327833MTL04023\_5
- Moschkovich, J. (2007). Using two languages when learning mathematics. *Educational studies in Mathematics*, 64(2), 121–144. DOI: 10.1007/s10649-005-9005-1
- Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, 6(1), 45–57.
- Norén, E. (2015). Agency and positioning in a multilingual mathematics classroom. *Educational Studies in Mathematics*, 89(2), 167–184. DOI 10.1007/s10649-015-9603-5
- Norén, E., Ramsfeldt, S., & Österling, L. (2016). *Tentative evaluation of the module Language in mathematics*. Unpublished evaluation, built on interviews, surveys and observations in classrooms. Stockholm University.
- Novak, J. D. (1990). Concept maps and Vee diagrams: Two metacognitive tools to facilitate meaningful learning. *Instructional science*, 19(1), 29–52.
- OECD (2010). *OECD Reviews of Migrant Education - Closing the Gap for Immigrant Students: Policies, Practice and Performance*. Retrieved the 17th of August 2016 at: <http://www.oecd.org/edu/school/oecdreviewsofmigranteducation-closingthegapforimmigrantstudentspoliciespracticeandperformance.htm#W>
- Österholm, M., & Bergqvist, E. (2013). What is so special about mathematical texts? Analyses of common claims in research literature and of properties of textbooks. *ZDM*, 45(5), 751–763. DOI: 10.1007/s11858-013-0522-6
- Prediger, S., Clarkson, P., & Bose, A. (2016). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, . . . M. Villavicencio Ubillús (Eds.), *Mathematics education and language diversity: The 21st ICMI study*, pp. 193–215. Heidelberg: Springer.
- Ramböll (2015). *Delutvärdering Matematiklyftet Läsåret 2014/15*. Stockholm. Retrieved the 15th of August 2016  
[https://www.skolverket.se/polopoly\\_fs/1.249368!/Slutlig%20version%20delutv%C3%A4rdering%20Matematiklyftet%20150904%20Pdf.pdf](https://www.skolverket.se/polopoly_fs/1.249368!/Slutlig%20version%20delutv%C3%A4rdering%20Matematiklyftet%20150904%20Pdf.pdf)
- Regeringen [the Government] (2012). *Government decision I 44*. Stockholm: Utbildningsdepartementet [Department of Education].
- Rezat, S., & Rezat, S. (2017). Subject-specific genres and genre awareness in integrated mathematics and language teaching. *Eurasia Journal of Mathematics, Science and Technology Education*, 3585–3606. doi: 10.12973/eurasia.2017.00805a
- Rose, D., & Martin, J. R. (2012). *Learning to write, reading to learn: Genre, knowledge and pedagogy in the Sydney School*. Sydney: Equinox.
- Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective*. Mahwah, NJ: Lawrence Erlbaum Associates Inc.
- Schleppegrell, M. (2007). The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review. *Reading & Writing Quarterly*, 23, 139–159. DOI: 10.1080/10573560601158461



- Schleppegrell, M. J., & O'Hallaron, C. L. (2011). Teaching academic language in L2 secondary settings. *Annual Review of Applied Linguistics*, 31, 3–18. DOI: <https://doi.org/10.1017/S0267190511000067>
- Schnellert, L. M., Butler, D. L., & Higginson, S. K. (2008). Co-constructors of data, co-constructors of meaning: Teacher professional development in an age of accountability. *Teaching and Teacher Education*, 24(3), 725–750. DOI: 10.1016/j.tate.2007.04.001
- Short, D. (2013). Training and sustaining effective teachers of sheltered instruction. *Theory Into Practice*, 52(2), 118–127. DOI: 10.1080/00405841.2013.770329
- Short, D., & Echevarria, J. (2004). Teacher skills to support English language learners. *Educational leadership*, 62(4), 8–13.
- Short, D., & Echevarria, J. (2016). *Developing academic language with the SIOP Model*. Boston: Pearson Allyn & Bacon.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher* 15(2), 4–14.
- Simon, M. (2014). Hypothetical Learning Trajectories in Mathematics Education. In S. Leman (Ed.), *Encyclopedia of Mathematics Education*, pp. 272–275. The Netherlands: Springer.
- Simon, M., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: an elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning* 6(2), 91–104. DOI: 10.1207/s15327833mtl0602\_2
- Sjöberg, G. (2006). *Om det inte är dyskalkyli-vad är det då? En multimetodstudie av eleven i matematikproblemet ur ett longitudinellt perspektiv*. Doctoral dissertation, Umeå universitet.
- Skolverket [National Agency for Education] (2004). Nationella utvärderingen av grundskolan 2003 - Huvudrapport - svenska/svenska som andraspråk, engelska, matematik och undersökningen i årskurs 5, Skolverkets rapport 251.
- Skolverket [National Agency for Education] (2011). *Kommentarmaterial till kursplanen i matematik*. Stockholm: Skolverket.
- Skolverket [National Agency for Education] (2012). Matematiklyftet. Beslut 2012-08-24. Retrieved from [http://www.skolverket.se/polopoly\\_fs/1.204676!/Menu/article/attachment/Programbeskrivning\\_Matematiklyftet240802013.pdf](http://www.skolverket.se/polopoly_fs/1.204676!/Menu/article/attachment/Programbeskrivning_Matematiklyftet240802013.pdf)
- Skolverket (2016a). Video: *Skolvägen, texten i uppgiften analyseras*. Retrieved from [https://matematiklyftet.skolverket.se/matematik/faces/training/newlink8190/newlink6604?\\_afLoop=565647300864336&\\_afWindowMode=0&\\_adf.ctrl-state=9gtodxui4\\_144](https://matematiklyftet.skolverket.se/matematik/faces/training/newlink8190/newlink6604?_afLoop=565647300864336&_afWindowMode=0&_adf.ctrl-state=9gtodxui4_144)
- Skolverket [National Agency for education] (2016b). *Redovisning av Regeringsuppdrag*. Retrieved from <http://mb.cision.com/Public/481/2130927/aa1e7a4e159cb8ea.pdf>
- Smit, J. (2013). *Scaffolding language in multilingual mathematics classrooms*. Utrecht University: Utrecht.
- Smit, J., & van Eerde, D. (2011). A teacher's learning process in dual design research: learning to scaffold language in a multilingual mathematics classroom. *ZDM*, 43(6–7), 889–900. DOI: 10.1007/s11858-011-0350-5
- Timperley, H., & Phillips, G. (2003). Changing and sustaining teachers' expectations through professional development in literacy. *Teaching and Teacher Education* 19(6), 627–641. DOI:10.1016/S0742-051X(03)00058-1

- Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2008). Teacher professional learning and development. *Educational Practises Serie - 18*. Brussels: The International Academy of Education (IAE).
- Turkan, S., de Oliveira, L., Lee, O., & Phelps, G. (2014). Proposing a knowledge base for teaching academic content to English language learners: Disciplinary linguistic knowledge. *Teachers College Record*, 116(1), 1-30.
- Van den Akker (2003) Curriculum perspectives: An introduction. In J. van den Akker, W. Kuiper, & U. Hameyer (Eds.), *Curriculum landscapes and trends*, pp. 1-10. Dordrecht: Kluwer Academic Publishers.
- Van den Akker, J. (2010). Building bridges: How research may improve curriculum policies and classroom practices. *Beyond Lisbon*, 201(0), 175-195.
- van Eerde, H. H. A., & Hajer, M. (2008). The integration of mathematics and language learning in multiethnic schools. In M. Cesar, & K. Kompulainen (Eds.) *Social Interactions in multicultural settings*, pp. 269-296. Rotterdam: Sense Publishers.
- Vogt, M. E., Echevarria, J., & Short, D. (2010). *The SIOP Model for Teaching English Language Arts to English Learners*. Boston: Pearson Allyn & Bacon.
- Wenger, E. (1998). *Communities of Practice. Learning, Meaning and Identity*. Cambridge: Cambridge University Press.
- Wiseman, A. W. (2008). A culture of (in) equality? A cross-national study of gender parity and gender segregation in national school systems. *Research in Comparative and International Education*, 3(2), 179-201. DOI: 10.2304/rcie.2008.3.2.179

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## APPENDIX

**Translation of self-reflection inventory instrument from the *Language in Mathematics* module**

Min matematikundervisning [*My mathematics teaching*]

Exempel på hur jag gör det [*Examples of how I do*]

- 1 Jag gör det matematiska innehållet begripligt genom att utgå ifrån elevernas erfarenheter och förkunskaper. [*I make the mathematical content comprehensible by departing from students' experiences and prior knowledge*]
- 2 Jag främjar aktiv språkanvändning genom att skapa tillfällen för eleverna att omväxlande tala, läsa, skriva och lyssna, under en lektion och under en serie lektioner. [*I promote the active use of language by creating opportunities for students to alternately speak, read, write, and listen, during a lesson and for a series of lessons*]
- 3 Jag planerar för att eleverna ska få syn på samband, likheter och skillnader, mellan matematikspråk och vardagsspråk. [*I plan for students to get sight of the connections, the similarities, and differences between the mathematical language and everyday language*]
- 4 Jag planerar för att ge eleverna många tillfällen att använda de olika delarna av matematikspråket. [*I plan to give students many opportunities to use different aspects of mathematics language*]
- 5 Jag ger exempel på framgångsrika strategier för att tolka matematiska problemtexter. [*I give examples of successful strategies to interpret mathematical problem texts*]
- 6 Jag uppmärksammar språkliga aspekter när jag formulerar syftet med mina matematiklektioner. [*I pay attention to linguistic aspects when I formulate the purpose of my math lessons*]
- 7 Jag organiserar aktiviteter för att få syn på elevernas förkunskaper inom ett område. [*I organise activities to get hold of students' prior knowledge in a mathematics area*]
- 8 Jag planerar aktivt en varierad språklig stöttning. (Makrostöttning) [*I actively plan a varied linguistic scaffolding. (Macro scaffolding)*]
- 9 Jag anpassar den språkliga stöttningen medan undervisningen pågår. (Mikrostöttning) [*I adapt linguistic scaffolding while teaching is in progress (Micro scaffolding)*]
- 10 Jag planerar aktiviteter så att alla elever ges möjlighet till muntlig interaktion. [*I plan activities so that all students are given the opportunity for verbal interaction*]

- 11 Jag planerar undervisningen så att eleverna utvecklar den matematiska kvalitén i sina samtal, under en lektion och under en serie lektioner. [*I plan teaching so that students develop the mathematical quality of their talk during a lesson and for a series of lessons*]
- 12 Jag uppmärksammar språkliga drag i olika matematiktexter, såsom faktarutor, typexempel, problemlösningstexter och redovisningar. [*I pay attention to linguistic features in different mathematics texts such as facts, typical examples, problem solving texts, and presentations*]
- 13 Jag ger eleverna möjlighet att producera olika sorters matematiska texter, tillsammans med mig och enskilt. [*I give students the opportunity to produce different kinds of mathematical texts, along with me and individually*]