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TORPEDO, A DIGITAL LEARNING ENVIRONMENT FOR DEVELOPING MATHEMATICAL PROBLEM-SOLVING ABILITY IN PRIMARY TEACHER EDUCATION

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Torpedo is a digital learning environment for developing mathematical problem-solving ability through self-study for pre-service teachers in primary teacher education. To achieve this, Torpedo supports and challenges pre-service teachers' reflection during and after solving non-routine mathematics problems. To investigate the feasibility of the Torpedo approach, 271 pre-service teachers used Torpedo during one month in a pilot study. They used and evaluated Torpedo's reflective elements differently. The results varied from pre-service teachers who experienced that reflection really contributed to the development of their problem-solving ability, to pre-service teachers who hardly reflected. The last group consisted of those who found the problems too difficult to reflect upon and those who used Torpedo to prepare for the National Mathematics Test and preferred to do so by drill and practice. As a conclusion, the study provides clues for improving Torpedo so that it invites more reflective self-study behaviour. For pre-service teachers who consider reflection valueless, however, self-study in a digital learning environment may be insufficient to change this attitude.

INTRODUCTION

Modern society requires coping with new problems and unexpected situations, in which using standard approaches is insufficient. In complex circumstances citizens and professionals need mathematical problem-solving ability. This enables them to construct suitable problem approaches for unknown, so-called non-routine mathematical problems (Pólya, 1990; Selden et al., 2000; Drijvers, 2015). It is important that teachers support their students in developing mathematical problem-solving ability and in doing this they have to be good problem solvers themselves, because that is one of the qualities they need to understand their students' problem approaches. This is why problem solving is an important focus in Dutch primary pre-service teacher education, and why it is assessed with a mandatory National Mathematics Test.

However, teacher educators are provided limited class time supporting the development of pre-service teachers' problem-solving ability. Pre-service teachers have to spend many hours on self-study as well. In this paper *self-study* is considered the situation in which learners are studying independently, outside the classroom without guidance. This interpretation of self-study only slightly refers to self-education, self-directed or self-regulated learning. In self-study pre-service teachers are invited to work in a recommended way with available digital study materials, but are free to disregard suggestions for use and employ the materials in a self-chosen way fitting their own needs. With regard to the recommended way of studying to develop problem-solving ability, reflection is the most important characteristic. If pre-service teachers want to develop their problem-solving ability it is valuable that they, while working on non-routine mathematical problems, during and after this work reflect on what they are doing or have done.

Reflecting here means reconsidering, monitoring and evaluating your work, and linking new knowledge to existing knowledge (Van Streun, 2001; Ambrose et al., 2010). Reflecting on mathematical problems has a cognitive component, – reflecting on the problem structure and the problem approaches (Gravemeijer, 1994) – and a metacognitive component, reflecting on the problem-solving process (Pólya, 1990; Verschaffel et al., 1999). Reflecting on the problem structure means the student decontextualizes the problem and relates it to problems with a similar mathematical problem structure. Doing so, the student realizes that he has not solved just one isolated problem, but one that represents a group of problems with a certain mathematical structure. Reflecting on problem approaches means the student compares different problem approaches and generalizes this into mathematical knowledge that can be applied in multiple situations (Gravemeijer, 1994). The student discovers how he can adjust the problem approach used, replace it with another one or relate it to one or more other problem approaches. Reflecting on the problem-solving process means the student evaluates the problem solving steps taken and the heuristics used while working on the problem (Pólya, 1990; Verschaffel et al., 1999). By looking back at the problem-solving process the student becomes aware of what he can do when he has to solve related non-routine mathematical problems. Heuristics are more general problem approaches like drawing a picture, making a list or a table or guessing and checking (Elia et al., 2009).

To reflect cognitively and metacognitively during and after working on a non-routine problem, students can use the four-step problem-solving approach of Pólya (1990). During the first step one tries to understand the problem by reformulating it or visualizing it schematically, then one chooses a problem approach that may be effective, thereafter one uses this problem approach to solve the problem and finally one checks and interprets the answer, which can lead to returning to earlier steps. In the final step the problem approach is evaluated and investigated to see whether and how it can be used in other situations. For developing problem-solving ability it is important that the four steps are used consciously and explicitly in education (Van Streun, 1989). It is also important that students can use and develop their cognitive and metacognitive knowledge and skills simultaneously (Mevarech & Kramarski, 1997; Hohn & Frey, 2002; Jacobse, 2012). Moreover, reflective problem solving can only take place successfully if the non-routine problems match the student's mathematical knowledge and skills (Drijvers, 2015). Reflecting during and after solving a problem not only contributes to the development of problem-solving ability, but students also become aware that problem solving has little to do with remembering and reproducing ready-made problem approaches per type of problem (Van Streun, 2014). They experience that doing mathematics is not using a fixed set of standard procedures, but that it is about knowledge construction (Bor-de Vries & Drijvers, 2015).

During class time the teacher educator strives to support and challenge pre-service teachers as much as possible to reflect on their problem-solving work. However, during self-study pre-service teachers are supposed to do the reflecting themselves. Kool and Keijzer (2018) explored student behaviour during mathematical self-study and discovered that pre-service teachers, when solving non-routine problems on their own hardly reflect on their work. They consider it not helpful, difficult, confusing and time-consuming. This implies that they probably do not develop their problem-solving ability during self-study. Pre-service teachers need support and challenge to reflect on their maths work during self-study, however regular study materials don't provide this. Therefore, we built a digital mathematical learning environment called Torpedo that offers these reflective support and challenge.

DESIGN OF THE TORPEDO ENVIRONMENT

Torpedo is a digital learning environment for self-study containing non-routine mathematics problems and various reflective elements. Torpedo provides (meta)cognitive reflective support by presenting expert reflections and (meta)cognitive reflective challenge stimulating to reflect yourself.

Torpedo contains ten problem groups, each consisting of five non-routine mathematics problems. Below each problem a problem-solving step-by-step path is shown, labelled: Understand the problem; Activate prior knowledge; Use hints; Enter your answer; Reflect on the problem structure, the problem approaches and the problem-solving process. This step-by-step path is based on the four-step problem-solving approach by Pólya (1990). For the main problem of each problem group, the problem-solving step-by-step path is fully worked out (figure 1), providing pre-service teachers as much support as they want.

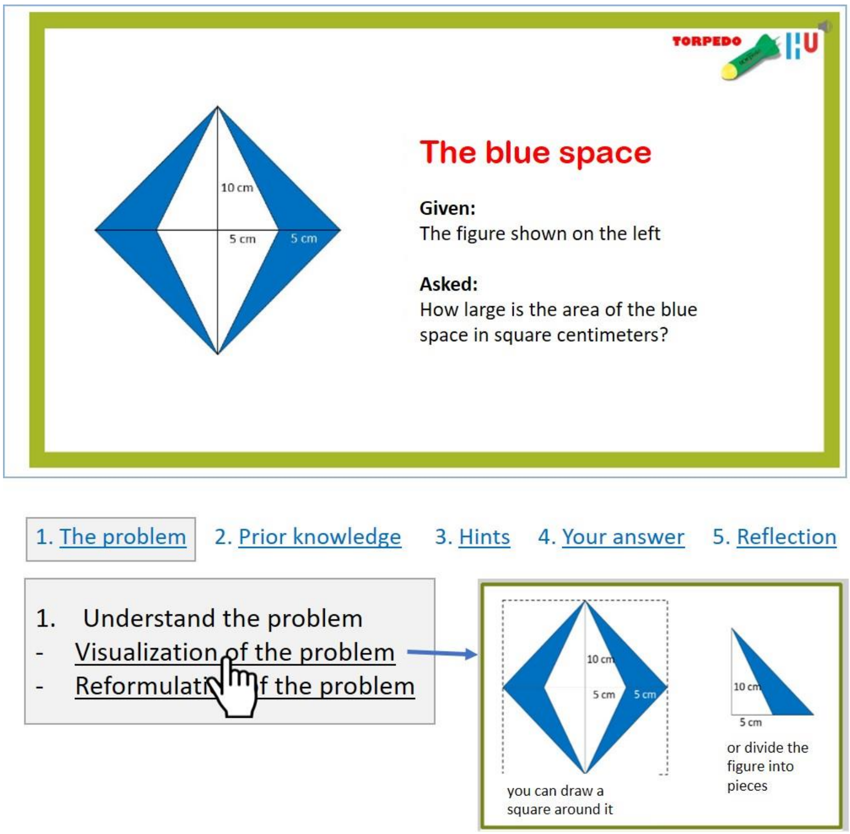

Figure 1: An example of a main problem in Torpedo provided with reflective support.

Working on the main problem, pre-service teachers can use this support. Moreover, they are supported by experts' reflections on the problem structure, problem approaches and the problem-solving process in six video clips. Each clip is followed by a question that enables pre-service teachers to check whether they understood the expert reflection. This question stimulates reflection. The main problem is followed by four so-called follow-up problems that are related to the main problem but still non-routine. Here the steps of the problem-solving step-by-step path are shown again but this time they are not connected with supportive elements. This means that here some meta-cognitive support is provided by the titles of the steps, but this time pre-service teachers have to reflect on the problems themselves. To encourage this an online discussion forum is added to two of the four follow-up problems where pre-service teachers can work together, asking for or offering help. Providing assistance to or understanding given assistance from fellow pre-service teachers requires reflection. Moreover, offering help and understanding other problem approaches is a useful activity for future teachers. One of the four follow-up problems contains primary school student work. Pre-service teachers have to reconstruct and evaluate student's problem approaches. Doing so demands reflecting on uncommon problem approaches. Moreover, this kind of non-routine problems indicates problem-solving ability is needed in pre-service teachers' future practice.

Pre-service teachers will be extra motivated to work in Torpedo, because the problems in the learning environment are at the National Mathematics Test's level, enabling them to use Torpedo in preparing for this test.

In investigating how pre-service teachers use Torpedo's reflective elements during self-study, we answer the following research question: To what extent do pre-service teachers study reflectively during self-study in Torpedo and how can their study behaviour be explained?

METHOD

In a pilot study, 271 pre-service teachers from 12 Dutch primary teacher education institutes used Torpedo from December 15, 2018 to January 15, 2019. Among them were 213 full-time and 43 part-time pre-service teachers and 15 pre-service teachers in an accelerated program. Accelerated means that these pre-service teachers try to accomplish their study in less time than the nominal four years.

Data included log data concerning the amount of time pre-service teachers spent in Torpedo, problems they tried to solve, correctly solved problems, contributions to discussions and answers on reflective questions. Additional data consisted of 181 responses to the after-pilot questionnaire. This questionnaire was developed to investigate the qualitative, reflective use of Torpedo and to find explanations for this use.

This questionnaire, consisting of 5-point Likert scale questions and open questions, is based on seven indicators of reflective study behaviour (table 1) derived from the reflection-supporting and -stimulating characteristics of Torpedo. Participants were invited to describe their behaviour with regard to these indicators and to evaluate Torpedo's elements and characteristics.

RESULTS

On average, the pre-service teachers spent 4.77 hours (SD = 5.82) in Torpedo and solved 12.4 problems (SD = 13.7). Time-on-task showed great variation and ranged from less than one hour to 37 hours, in which 46 pre-service teachers did not solve any problem, while 13 solved all 50 problems.

There was no correlation between the time pre-service teachers spent in Torpedo and the number of solved problems. Some pre-service teachers who entered little or no answers indicated in the questionnaire that they had collaborated with a peer and entered their answers on just one computer. Pre-service teachers who solved many problems in little time didn't spend much time on reflecting, but pre-service teachers who spent a lot of time solving problems didn't necessarily show reflective study behaviour. Some pre-service teachers indicated that the problems were too hard to solve in a quick way. The seven indicators of reflective study behaviour provide insight into the way in which and the extent to which pre-service teachers reflected in Torpedo. Table 1 shows the percentage of pre-service teachers who solved at least one problem and showed at least once the distinguished indicators of reflective study behaviour. These results are derived from the log data and /or from the answers to the questionnaire.

Indicators of reflective study behaviour	Pre-service teachers
1. The pre-service teachers use the worked out problem-solving step-by-step path that goes with the main problem.	89%
2. The pre-service teachers answer the reflective questions that go with the reflective videos.	54%
3. The pre-service teachers work on one or more follow-up problems after they have worked on the main problem.	85%
4. The pre-service teachers, if they get stuck with the follow-up problems look for help within Torpedo.	25%
5. The pre-service teachers work on problems with student work.	68%
6. a. The pre-service teachers contribute to discussions.	14%
6. b. The pre-service teachers read contributions in discussions.	85%
7. The pre-service teachers take notes while working in Torpedo.	81%

Table 1: Percentage of N = 271 pre-service teachers showing indicators of reflection at least once

Indicators 1, 3, 6b and 7 score remarkably high. Indicators 1 and 6b refer to reflections from experts and fellow pre-service teachers. In these cases, the pre-service teacher can read these reflections, but doesn't necessarily have to reflect himself. Indicator 3 relates to follow-up problems. Here the problem-solving step-by-step path is shown but in contrast to the main problem no further information is given. Now the pre-service teacher is challenged to reflect himself. However, it is not certain whether he actually does so, because it is possible to solve follow-up problems without reflecting, that is, without relating them to the main problem of the group. Similar remarks may be made on indicator 7. It may be assumed that pre-service teachers think about what they want to note during their working in Torpedo, but it is conceivable that they simply copied problem approaches from the learning environment without any reflection. Indicators that convincingly show that a pre-service teacher has actively reflected, are indicators 2, 4, 5 and 6a. It is impossible to perform these activities without reflection. These reflection-stimulating indicators are relatively less often observed than the reflection-supporting indicators. The least observed indicators are: 4. If the pre-service teachers got stuck with the follow-up problems, they looked for help within Torpedo (25%), and 6a. The pre-service teachers contribute to discussions (14%). Compared to the other indicators these seem the most demanding tasks. The least observed indicators are related to activities that require reflection on a high level.

To find some explanation for this behaviour, pre-service teachers were invited to evaluate Torpedo's elements and characteristics in the questionnaire, which was completed by 181 of the 271 pre-service teachers. Results show that the majority of pre-service teachers appreciated the reflection-supporting elements added to the main problem, such as the support given in the problem solving step-by-step path and the reflective videos (M = 4.11, SD = 0.90 in a 5-point Likert scale). They generally did not appreciate the lack of support in the follow-up problems that were intended to encourage them to actively reflecting themselves (M = 2.16, SD = 1.19). Concerning the learning outcomes of their working in Torpedo we asked them to score on the 5-point Likert scale on the statement 'I know better what I can try or do if I have to solve a new mathematical problem.' Of the 181 respondents, 82 scored 4 or 5 on this question, meaning that these pre-service teachers experienced that working in Torpedo helped them to develop their problem-solving ability, 40 respondents scored 1 or 2 and have hardly experienced any contribution from Torpedo to their problem-solving ability. The open question that was connected to this statement was answered by 44 pre-service teachers, 47% of whom were convinced that thanks to Torpedo, they dealt with the problems in a more structured, thoughtful, calm and systematic way during the National Mathematics Test: "After studying with Torpedo, I worked more relaxed on the problems in the test.", "I was much more aware of the thinking steps to be taken.", "Torpedo has taught me to think deeper about each problem." 43% of these respondents claimed that Torpedo has brought them little or nothing. They gave little explanation to their answer, but we suppose that abandon reflecting has contributed to the lack of experienced learning outcomes. We have no data to confirm this assumption.

The open question on motivating and demotivating aspects of Torpedo was answered by 71 pre-service teachers. Out of them, 34 (47.9%) appeared demotivated by the lack of support in the follow-up problems, and 12 of these 34 respondents (38.2%) explained that reflecting in Torpedo was too difficult for them. The problem difficulty made them need more support to solve the problem and reflect: "Torpedo has brought me nothing because it was really too difficult." 8 of these 34 respondents (23.5%) wanted more instruction because they rejected reflection as being cumbersome, time-consuming and ineffective. They were not convinced that reflection can contribute to their problem-solving ability: "I want to practice more problems in a shorter time.", "That solving a problem takes more than half an hour is very demotivating.", and "I like to solve a large amount of problems and want to see afterwards if I have solved them properly." They seemed to ignore Torpedo's aim being developing problem-solving ability. They instead focused on preparing for the National Mathematics Test and seemed to consider drill and practice the best way to do so. Apparently they think that learning mathematics is memorizing problem approaches, and problem solving is relating problem approaches and problems only. This view might hinder the development of problem-solving ability (Crawford, 1994; Kloosterman, 2002).

CONCLUSION AND DISCUSSION

In this study, we set out to address the research question: To what extent do pre-service teachers study reflectively during self-study in Torpedo and how can their study behaviour be explained? We conclude that Torpedo can support and stimulate reflective study behaviour during self-study and in this way may support the development of pre-service teachers' problem solving ability. Pre-service teachers who reported that Torpedo supported them in dealing with test problems in a more structured, thoughtful and systematic way apparently benefited from Torpedo in simultaneously developing their cognitive and metacognitive knowledge and skills. The individual differences amongst the pre-service teachers in their use of and appreciation for Torpedo, however, are huge. Some pre-service teachers did hardly reflect or not at all. They explained that they found the problems too difficult to do so, and/or considered reflecting too cumbersome and time-consuming, and/or were not convinced that reflecting would contribute to the development of their problem-solving ability.

In interpreting these conclusions, we conjecture that the participants who studied reflectively in Torpedo and judged this to be effective share three characteristics: to them, the problems in Torpedo were at the appropriate level, they were willing to invest time and effort in reflective study, and they were convinced that reflection during and after solving non-routine problems would contribute to the development of their mathematical problem solving ability. Other pre-service teachers, who did not share these characteristics nevertheless worked in Torpedo, but usually not in a reflective way. Rather, they used Torpedo as a conventional practice environment.

These major differences in the use of and appreciation for Torpedo may have been caused by the Torpedo's indirect aim. The learning environment focuses first and foremost on the development of problem-solving ability and, second, on the preparation for the National Mathematics Test, which is why Torpedo consists of problems at the test level. This may have motivated a number of pre-service teachers to participate in the Torpedo trial while not being convinced of the usefulness of reflecting in mathematics learning. Moreover, non-routine math problems at the National Mathematics Test level are quite difficult for pre-service teachers. For some participants the difficulty level of the non-routine problems did not match their mathematical knowledge and skills, as we saw before, which may have hindered reflective problem solving (Drijvers, 2015).

How to improve Torpedo? A first way of doing so is by focusing exclusively on the development of problem-solving ability. However, if the link with the National Mathematics Test would disappear, pre-service teachers might be less motivated to work in Torpedo. Second, Torpedo might become more effective through a gradual increase in the difficulty of the problems and a gradual reduction of the reflection-supporting elements. Third, Torpedo can make the pre-service teachers (more) aware of its primary aim, which means the development of mathematical problem solving ability and try to convince them that reflection is an effective way to reach this aim.

The latter point, to convince pre-service teachers of the importance of reflection, will be the most challenging, because it has to do with pre-service teachers' view of mathematics and mathematics learning. If they consider problem solving as recognizing the problem and using the right trick to solve it, and if they consider mathematics learning as memorizing problems and problem approaches, then they are unlikely to see the usefulness of reflection. Even video clips showing experts using reflection to construct problem approaches for non-routine problems cannot convince them. Inviting them to study reflectively in a digital learning environment like Torpedo is of little use. Providing them, while working on the follow-up problems, with all the support and instruction they are asking for is probably not helpful either. Pre-service teachers need regular encouragement to reflect (Nelissen, 1999). A digital learning environment can provide this, but such an environment is unlikely to change the beliefs about learning mathematics that some pre-service teachers have. To achieve this, pre-service teachers probably need fellow pre-service teachers and teacher educators who can show and make them experience how reflection can contribute to the development of their problem-solving ability. Blended use of Torpedo could be helpful here, where peers and teacher educators form a scaffold for reflection. This scaffolding might result in new opinions about learning math and reflective self-study behaviour. If Torpedo through blended use could encourage more pre-service teachers to study reflectively, new research might focus on what this contributes to the development of their problem-solving ability.

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